Exploring Curves

Notes:

- Answers to questions within the lesson are highlighted in yellow.
- An answer sheet corresponding to the worksheet follows the lesson plan
- Potential roadblocks: none
- Front-loading vocabulary: none – introduce as we proceed

Materials

- Exploring Curves Worksheet: 3 pages; I print page 1 and 2 back-to-back, and do worksheet 3 as separate page NOt stapled to others.
- Time note: Worksheets 1 and 2 can be completed in a 30 minutes without really rushing. Worksheet 3 requires about 15 minutes to do with discussion. You can use this as a separate lesson.

1. What is topology?

This lesson explores lines and curves, and the concept of inside and outside. These are concepts from an area of mathematics called topology.

Mathematics has many different areas you can study. We've all studied arithmetic which looks at how addition, subtraction, multiplication and division work. Geometry studies shapes, like triangles and circles and squares. Topology studies how things are connected, and doesn't care about the shape. You can twist, bend and stretch a figure, as long as you don't break any lines. We'll explore some of these ideas with a few lessons over the next few weeks.

2. Instructions for this lesson

In a minute I'm going to hand out a paper with lots of fun problems. I need you to write you name at the top and put your pencil down and NOT do any of the problems. The problems look easy and you may be able to do them without any instructions, BUT you will have more fun and learn more if you do them with me.

For each problem, I'm going to do something on the board, then tell you to do those problems. Then I'll do more on the board and continue.

When we turn the page over there are four mazes. Please do NOT do them until I discuss them with you. Each one has a different puzzle piece and I'll go over them with you.

3. open and closed curves

We're going to look at diagrams drawn on a flat surface, like our blackboard.

Note: For each concept below, draw a couple examples on the board and tell/ask the students questions. I do this as an interactive oral start.
See answer sheet for sample curves.

Draw an open curve and say "this is open" and then a closed curve and say "this is closed." Then draw another open curve and
ASK: What do you think, is this open or closed?
Repeat for a few curves; ask class to classify.

Write the digits 0, 1, 2,..9 on the board.
ASK: Which digits are open curves and which are closed? [the digis 0 and 8 are closed curves and all the rest are open]

Give out handout and have students look at the side that says **open or closed**?

Tell class to mark which curves are open and which are closed.
[answers: o, c, o, o, c, c]

4. simple and non-simple curves

A simple curve is a curve that does not cross itself. (Note: I draw a long scribbly curve to be sure it does not cross itself.)
A non-simple curve does cross itself. (I draw a very simple loop)

On the handout, mark which curves are simple and which are non-simple.
[ans: s, ns, s, s, s, ns]

5. polygons

A special group of simple closed curves is called polygons. Poly means many and gon means side, so a polygon is a many sided figure. A polygon is a simple closed curve **where all the sides are line segments** (straight, no curving allowed).

Which of the next six figures are polygons?
Discuss why each is or is not a polygon.

No, not closed	No, not simple – 2 triangles with common vertex)	No, not just line segments	yes	No, not simple – triangle and quadrilateral	Yes, 48 sides; it appears curved but isn't

6. Draw sample curves

Have students draw sample of each type of curve:

- Open simple
- Open non-simple
- Closed simple not a polygon
- Closed simple and a polygon

Walk round room and check work.
Put the four types on the board and have four students come up and draw their example.
Use student diagrams for next part of lesson

7. Exploring inside and outside – a maze

Let's look at the four figures drawn on the board.
Question: Does the open simple curve divide the blackboard into regions, that is, into a space inside and outside? [no]
Question: Does the non-simple curve divide the blackboard into regions, a space inside and outside? [yes]
Question: Does the closed simple not a polygon curve divide the blackboard into regions, a space inside and outside? [yes]
Question: Does the polygon divide the blackboard into regions, a space inside and outside? [yes]

For the simple open curve, if we have points A and B on the board and want to connect them. Can we do so without crossing the curve? [yes]

For the polygon, if we have a point C inside the polygon and D outside the polygon, can we connect them without crossing the curve? [no]

Now turn over your papers. We call each of these figures a **maze**!
ASK: Is the maze a simple closed curve? [yes]
Why? [because divides region on paper into exactly two regions and does not cross itself]
ASK: Is the maze a polygon? [Yes]
Why? [because made of only line segments]

8. Maze 1

ASK: Is point A inside or outside the maze? [outside]

How about the other points? Without using your pencil, can you tell which points are inside and which are outside the maze? Take answers and comments, then say "using your pencil, go ahead and figure out which points are inside and which outside."

Please shade quickly and lightly the **inside** of the maze. [Note: on answer sheet, only maze 1 is shaded.]

9. Maze 2

Before you use your pencil please look the maze over. Is point E inside or outside. If E is outside, what can you say about point F? point G, point H? Why? Now use your pencil to prove your answers.

Shade very quickly and lightly the **outside** of the maze.

10.Maze 3

Just look at points J, K, L. Can all three of them be inside the maze? Why?
Can all three be outside the maze? Why?
[since they are on two different sides of a line segment making up the maze, one must be inside while the other outside]

Draw a line from M to N. How many sides of the maze did you have to cross? [1]
Draw a line from M to O. How many sides of the maze did you have to cross? [4]
Draw a line from N to O. How many sides of the maze did you have to cross? [3]

11.Maze 4

Figure out which is inside and which is outside anyway you want.
Make up a hypotheses , that is a rule you can check, from what we did with the other mazes and test your ideas.

Ask the kids how they figured out which were inside and outside. [options include shading inside of maze, connect points, counting crossing of maze sides, just running finger along maze, and possibly others]

12.Page 3 Analyzing Inside and Outside

Please complete the first column on the table at the bottom of the page.

Point	Inside or Outside	# Paths Crossed From A
A	Outside	0
B	Inside	3
C	Outside	8
D	Inside	11

- Can you get from A to B without crossing any lines? So is A outside? How about B? [yes]
- Is C inside or outside? Can you go C to B without crossing any lines? [no]
- Put a new point E outside the maze and just to the left of the letter D.
- How many lines do we need to cross to get to D? [one, so D is inside the maze.]
- If we continue past D toward B and we cross just one line, we'd now be outside the maze. So crossing one line takes us inside and crossing two outside.
- Can you think of a rule that explains the number of paths you cross and being inside or outside? [Cross odd number of paths you are now inside; cross an even number of paths you are still outside]

13.Make a maze problem for your friends

Draw a maze, making it as challenging as you can, and put some points inside and outside and ask your friends which points are which. (Note: Your maze does not have to be only line segments.)

=== END OF LESSON PLAN ===