Exploring Traceable Figures

Notes:
- Answers to questions within the lesson are highlighted in yellow.
- Potential roadblocks:
- Front-loading vocabulary: vertex,

Materials
- Exploring Traceable Figures worksheet: 1 pages

1. Review of topology lesson 1 Types of Curves

Draw a sample of each curve:
- Open simple
- Open non-simple
- Closed simple not a polygon
- Closed simple and a polygon

Have students name these as they are drawn.

ASK: Does the open simple have an inside and outside? [no, we can say all outside]
ASK: Does an open non-simple have an inside and outside? [yes, where the curve crosses itself it forms a loop which would be considered "inside the curve."]
ASK: Can an open simple be made into a open non-simple by just moving the line around, without crossing itself? [no]
ASK: Does the closed simple not a polygon have an inside and outside? [yes]
ASK: Does an closed simple and a polygon have an inside and outside? [yes]
ASK: Can an closed simple not a polygon be made into a closed simple and a polygon by just moving the line around, without breaking the curve nor crossing itself? [yes]

2. Introducing the wonderful world of topology

Mathematicians are curious about all sorts of things and like to investigate ideas from many points of view. When they look at figures like circle, squares, triangles, they can talk about the length of the distance around the figure (the circumference for a circle and the perimeter for the polygons), or about how much space the figure covers, or area.

But there is another way to investigate these figures. In a previous topology lesson we talked about properties of being inside and outside a curve, or whether a curve was simple or non-simple. You may remember we said that topology was about looking at the properties as long as you don't break or add any parts of the figure.

In fractions, we say that two fractions are **equal** if they have the same value, such as 2/4 and ½.
In geometry, we say that two figures are **congruent** if they are the same shape and size. A 2x4 rectangle is congruent to a 4x2 rectangle.

In topology, we will say that two figures are **topologically equivalent** they can be reshaped one into the other without breaking any curves or adding any segments. A triangle, circle and square are all **topologically equivalent** because a circle can be reshaped into a square or a triangle, but not into a line as that requires breaking the circle.

We're going to explore that idea with some other curves today.

3. Traceability and the alphabet

Hand out papers – start with traceability challenge.
A letter is traceable if you can copy it without lifting your pencil from the paper and not going down any path more than once. We can go through a point or vertex more than once, but not down any path. Let's do some examples.

Note: for this lesson we are using Arial font letters. There are different ways to write the letters of the alphabet, but we're using what is on the paper only.
On your paper, just trace the letters with your finger so we can keep the letter for later use.
Question: Is the letter A traceable? Use your finger and try to trace it. [no, it isn't, so don't circle it.]
Question: Is B traceable? [yes]
Go ahead and circle all the letters that are traceable.

After few minute read the letters aloud; have the kids say "yes" or "no".

Q is usually a mixed response, so draw a large Q (using Arial font). You can say "In topology that the endpoints of the paths and points were paths intersect are important" so mark them with dots. This helps the kids see how to trace Q.

4. topologically equivalent

We said that in topology it is the properties that are important and not the way the figure looks. For example, suppose the letter C were made with a string. You can straighten the string and make an I. [draw both a C and I on the board].
The topological properties of these two figures are the same: they are both open simple curves and have two endpoints. They are topologically equivalent.

They are not topologically equivalent, to X which is an open non-simple curve with four endpoints.

Look at the table at the bottom of the handout. Let's start with the C group: there are 12 letters which are topologically equivalent to C: let's name a few. [C, G, I, J, L, M, N, S, U, V, W, Z]

Let's do one more example: what is a letter topologically equivalent to E? [E, F, T, Y]

Now you take a few minutes to try and find the letters which are topologically equivalent to each other.

Go over answers with class:

Letter	Topologically Equivalent Letters
A (2 total)	A, R
C (12 total)	C, G, I, J ,L, M, N, S, U, V, W, Z
D (2 total)	D, O
E (4 total)	E, F, T, Y
H (2 total)	H, K

	No Topologically Equivalent Letters
(4 total)	B, P, Q, X

=== END OF LESSON PLAN ===