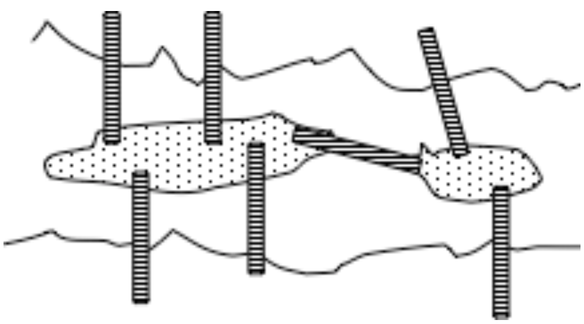
# Exploring Networks

Notes:
- Answers to questions within the lesson are highlighted in yellow.
- Potential roadblocks:
- Front-loading vocabulary: Even, odd

Materials
- Exploring Network Worksheet: 1 page

## 1. Introducing the wonderful world of networks

**Topology** is one of the newer areas of mathematics, and now one of the most important. It got its start in 1780s in town of Kongsberg, Germany. The town is on the Prequel River and there were 7 bridges. The newspaper offered a prize to whoever could walk cross all seven bridges and get back to the starting point, but they could only cross each bridge once.



The problem was solved by Leonard Euler. Euler developed the first ideas of what is now called **network theory**, which is extremely important area in the world today. Delivery truck drivers and computer designers use network theory to figure out the route of their trucks and to make the circuits in computers most efficient (as small or short as possible).

To solve the bridge problem we need to use the ideas of traceable figures. Let's investigate.

## 2. Review Traceability

Last lesson we looked at which letters where traceable. We said a letter is traceable if you can copy it without lifting your pencil and not repeat any path. We found that 9 letters like A, H, and Y where not traceable, but that 17 letters like B, D and Q, were. Today we're going to investigate some figures to find out which are and which are not traceable, and try to find a rule to help us solve the bridge problem.

## 3. Networks

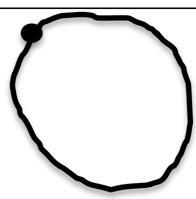
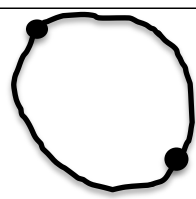
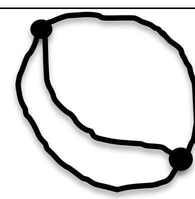
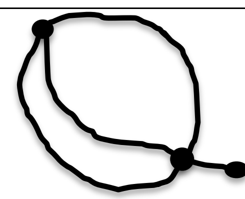
What is a network?
A network is any curve you can draw on a paper. That is, the word network is used to mean any open or closed, simple or non-simple curve.
Each letters of the alphabet is a network.

To help us describe a network, there are three words mathematicians use:

- The points are called VERTICES (one is a vertex)
- The lines connecting the vertices are called PATHS
- The spaces created by the paths are called REGIONS

All the curves we've drawn in the past, the open and closed, simple and non-simple, can each be called a network. They all have vertices, paths and regions.

Draw a couple of different networks, some with different number of regions: start with a circle and add diagonals, and additional loops.

|  |  |  |  |
|---|---|---|---|
| When you draw a loop, the starting and ending point is the vertex. This loop has 1 vertex, 1 path (the entire loop), and 2 regions: inside and outside. | We added one vertex, so now there are 2 vertices, 2 paths, and still 2 regions. By adding the second vertex we divide the original loop. | We added a path between the 2 vertices, which divided the inside region into two. So, now we have 2 vertices, 3 paths and 3 regions. | Adding a path also requires adding a vertex. Since this was an open curve, it does not further divide the outside region. We now have 3 vertices, 4 paths, and 3 regions. |

## 4. Networks and Traceability

Distribute the handout.

The handout has 9 networks drawn. Let's focus on the first six right now. Notice that the 9th is the Kongsberg bridge problem.

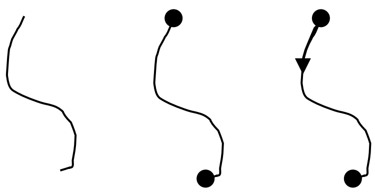
Please test the first six networks and let's see which are traceable.
Give class several minutes to do these networks and then stop to go over them.

Networks 1, 2, 3 and 4 are traceable.
Networks 5 and 6 are not.

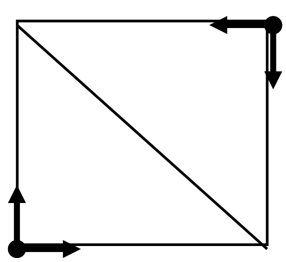
You can predict whether a network is traceable by counting the number of odd vertices, so let's investigate.

## 5. Even and odd vertices

To help with our investigation, we need to label the vertices of our networks as even and odd. We determine this by counting how many ways you can leave a vertex.

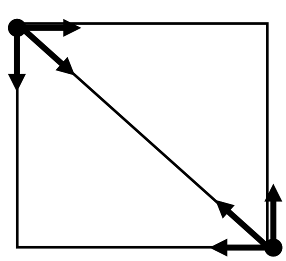


Here is a series of drawing showing a single path, and then the vertices marked as endpoints, and then an arrow showing that there is exactly one way to leave that vertex. We say that vertex is an odd vertex.



Now make a square and draw one diagonal.

The two vertices without a diagonal are even as there are two ways to leave each of those vertices.



The two vertices where the diagonal intersects are odd because there are three ways to leave the vertex.

To determine if a vertex is even or odd we count the number of ways to LEAVE the vertex.

Now go back to networks 1 through 6 and put a little circle around all the ODD vertices. Count them and put the numbers into the table. [Your table should look like this.]

| Net | Traceable | # odd |
|---|---|---|
| 1 | Yes | 0 |
| 2 | Yes | 0 |
| 3 | Yes | 2 |
| 4 | Yes | 2 |
| 5 | No | 4 |
| 6 | No | 4 |

What observation do you have? Can you now answer the question on the traceability rule?

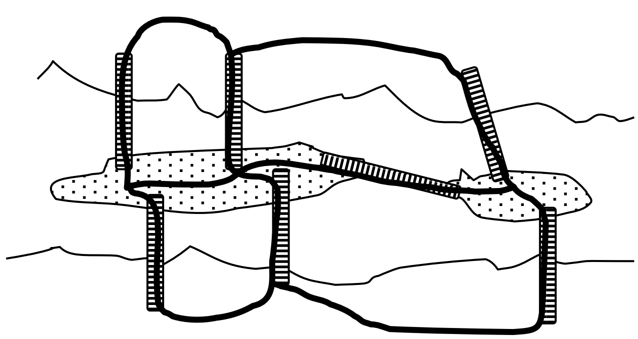
If there are _4_ or more odd vertices then you cannot trace the network without lifting your pencil!

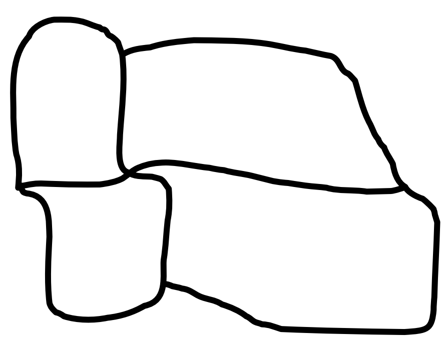
## 6. Predicating if you can trace a network

Now test your rule by counting the number of odd vertices in networks 7 and 8 and try to trace them.
What is the difference between the two networks? [The extra path in 8.]
Number of odd vertices? [Net 7 has 2 odd vertices and can be traced, but net 8 has 4 and cannot be traced.]
Why? [With even vertices, you can always leave and come back to a vertex. With odd, once you leave and come back, you cannot leave (or come back) again. ]

## 7. Kongberg Bridge Problem

To help solve the Kongsberg bridge problem turn the picture into a network by tracing over all the bridges and connect them all so that would be path that a person would walk to go over them all.





On the back of your paper draw the bridges as a network and do your analysis.
[there are four odd vertices so the network is NOT traceable]

## 8. Make your own – challenge your friends

Draw a couple of different networks, some with different number of regions: start with a circle and add diagonals, and additional loops.

Turn your paper over and make a couple of networks, one traceable and one not traceable. Have them trade papers. Have them make predications before attempting if it is traceable or not. Hint: to make a non-traceable network, draw one that is traceable and then add a path to create an odd vertex. You can add two or three of these kinds of paths.

Do worksheet asking to count number of even and number of odd vertices. What is the relationship between number of even or odd vertices and whether the network is traceable?

=== END OF LESSON PLAN ===