Exploring Surfaces: Mobius Strip!

Notes:

- Answers to questions within the lesson are highlighted in yellow.
- Potential roadblocks: following instructions; putting the letters on the strips and than matching them; may need to demonstrate more than once
- Front-loading vocabulary: vertices (corner) of a rectangle

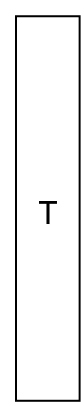
Materials

- No handout
- Two (2) strips of paper for each student (I cut 8 ½ x 11 into 1 x 11 inch strips).
- Pencil, scissors and we about 2 inches of tape to make loops.
- I made two demonstration strips that were double long and wide (about 22 long and 1.5 wide).
- Distribute the scissors, two strips and about 2 inches of tape (or tape ready for use) before starting.
- Be sure to have extra strips for the kids as they break them or tape them wrong.

1. What is topology? (same intro as other topology lessons)

We've explore some different idea from the area of mathematics called topology. Geometry studies shapes, like triangles and circles and squares. Topology studies how things are connected, and doesn't care about the shape. You can twist, bend and stretch a figure, as long as you don't break any lines. We found that in topology, a triangle, circle and square are all equivalent as they each are curves that divide a surface (a plane) into two regions, an inside and outside. Today we're going to find out what happen when you twist a paper strip.

2. Exploring the strips



I've made some extra large loops to show you what to do, but you need to do these steps yourself.

We're going to explore properties of some loops which we will form with paper strips.
Draw a picture of the strip (a rectangle) on the board.

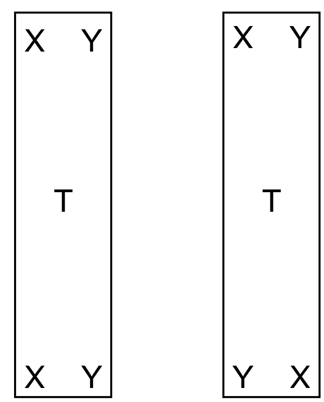
Look at your strip of paper.
ASK: What shape is your strip? [rectangle]
ASK: How many vertices does it have? [4]
ASK: How many sides or paths connecting the vertices? [4]
ASK: How many faces does the strip have? [2]

We also call these sides of a piece of paper. Notice how the word side is used in two different ways. Sides of a rectangle and sides of a piece of paper.

ASK: How could you prove to me there are two faces (sides)? [lots of acceptable answers such as touch both sides at same time or cannot draw a line from one to other without turning paper over.]

Put a T in the middle of one face (side) (we'll call it the top) and a B in the middle of the back.
Draw two strips on the board and mark the middle of each with a T.
ASK: If you were to draw a line starting at the T, could you get to the B without lifting your pencil? [no]

3. Preparing your strip construction



At the top of the face (side) with a T write the letters X Y.
On one strip at the bottom put the letters X Y.
On the bottom of the other strip put Y X.

4. Constructing the loop

Using the strip that has   and both top and bottom, make a loop so that the Xs and Ys are connected (on the same side of the strip; that is, X to X and Y to Y
Tape the strip to make a loop.
ASK: How many sides does our strip have? [two]
ASK: If you were to draw a line down the middle of your strip starting at T, would you ever get to B? Do it, draw the line. [no, they are on opposite sides of loop]
ASK: What do you get when you draw the line all the way round the loop? [back to the T]

5. Constructing the Mobius strip

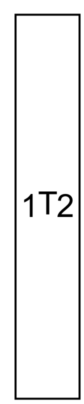
Please set aside your loop and let's explore the other piece of paper. We're going to make a loop again, but with a slight difference.
Now make a loop again by touching X to X and Y to Y, and tape.
ASK: What do you have to do to make the X's touch this time? [This is challenge for kids – tell them to be creative in figuring out how to do it. Think "outside the box." You have to put a twist into the paper.]

ASK: How many sides does this loop have? [Expect students to say "two"]
Let's investigate: Starting at the T, draw a line down the middle of your strip just like we did before.
ASK: What happens? [get to the B,on the "other side"!]
ASK: What happens if you keep going? [you get back to the number T]
ASK: What does this mean, that you get back to T? [The strip must have only one side!]

6. Mobius strip experiments

We're going to further challenge what appears as common sense reasoning with a couple of experiments.

On both your strips, put the numbers 1 and 2 on either side of the T and B, 1T2 and 1B2.



Are they on the same face (side) of the loop and Mobius strip? [yes]
We're not going to cut the loop down middle, along the line you use to connect the letters, like this…

Start the cut but stop after a few inches.

ASK rhetorical questions (no answers now): What do you think will happen? How many loops? Which numbers will be on which side?
Have the kids cut their loops and strips and discuss outcome.

The loop cuts into two distinct separate loops, and the number one is on one side and the number two is on the other for both loops.

For the Mobius strip becomes very long loop with some twists (actually two twists). If you start at the number one and trace a line, what number do you think you'll reach? Try it. You get to number 1 again!!!

What do you think would happen if I cut the strip again?

You can do this experiment at home.

7. Historical and usefulness note

What we call the Mobius strip was discovered by two different people in two different places in 1858. We call this an independent discovery, and it has happened many times in mathematics. Maybe you will make a discovery some day!

Interesting fact: Mobius strips are used with conveyor belts [do they know what these are?] so that the belt wears evenly on "both" sides.

=== END OF LESSON PLAN ===