Exploring Fractals

Notes:

- Answers to questions within the lesson are highlighted in yellow.
- Potential roadblocks: following directions; first time they need to be VERY careful on where they put the dots for the midpoints of the sides
- Front-loading vocabulary:

Materials:

- Exploring Fractals Worksheet: 2 pages
- Color pencils or crayons optional
- 40 minute lesson without table discussion
- 60 minute with table discussions
- Background information if you want to understand a bit more:
    - A simple to read and understand reference: Cynthia Lanius
    - http://math.rice.edu/~lanius/fractals/WHY/
    - Excellent visual introduction is PBS NOVA (only need to watch first five minutes):
    - http://www.pbs.org/wgbh/nova/physics/hunting-hidden-dimension.html

1. A Tree fractal

- Start with a V. The V is made with two line segments.

ASK: what is a line segment? [part of a line that is has two end points]

- Rule: Put a V at the end of each line segment
- Keep repeating this pattern.

What does the drawing look like?

We call this figure **self similar**, that is, each piece, no matter how closely you look at it, looks like the larger piece.

2. What is a Fractal

- A fractal is a figure that repeatedly follows the same rule and continually reproduces copies of its self.
- To make a fractal, start with any shape and make up any rule you want. Follows the rule over and over again making more copies of the same shape.

- Try it with a letter L and see what happens? (the fractal at the right is made from 7 of the L's at the left)
- We're going to do really interesting fractals in a few minutes. You can make more at home, but keep your starting shape simple.

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3. Fractals and Nature

- Objects in nature often look fractal in structure.
- Most objects in nature aren't formed with straight lines, squares or triangles, but of more complicated geometric figures.
- A fractal is "a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole," a property called self-similarity.
- Trees and flowers are self-similar, reproducing themselves endlessly.
- For example, think of a tree: Start with one branch, another splits off. From each of those we get a split. If there is only one split, the next time it splits, its only one split, not two or three, but one. That is, it is self-similar. Doesn't matter if you look at the biggest branch or the smallest, oldest or newest, it is self-similar.
- Other things in name like clouds, waves, and mountaintops, all exhibit self-similar patterns.

4. What's so hot about fractals?

- Most math you study in school is old knowledge. For example, in geometry we learn about circles, squares, and triangles and that was originally studied around 300 B.C., by a man named Euclid. [How many years ago was this? About 2300 years ago]
- Fractal, on the other hand is new, discovered in 1979 by Benoît B. Mandelbrot. He died in 2010. He needed to use supercomputers to help make his discovery.
- Fractals allow mathematicians to study things like
- Mandelbrot said that things typically considered to be "rough", a "mess" or "chaotic" by which he meant that clouds and plants don't seem to be well organized or follow a pattern. He showed that they do!
- There are fractal research projects currently being done all over the world as fractals help explain ideas which in past mathematics could not figure out

5. Fractals art

We're now going to do a couple of fractal examples that were discovered by mathematics about 100 years ago, before anyone knew that they were really fractals. We're going to create what is now called the Kock snowflake, discovered by the Swedish mathematician Helge von Koch in 1902, and we're going to start a fractal named after the Polish mathematician Wacaw Sierpiski who described it in 1915.

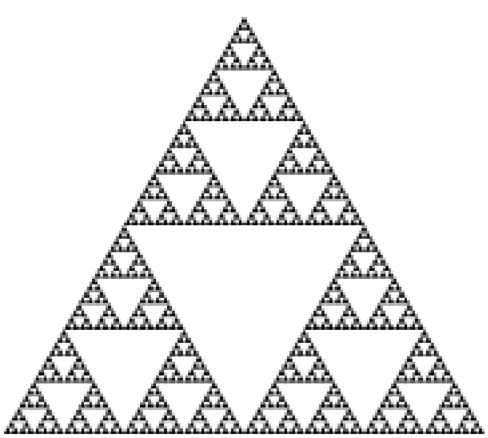
When doing fractal art, use a pencil and draw your lines lightly as you need to erase some parts of your drawing as you go along. I'm handing out some special paper with lots of equilateral triangles. (What's an equilateral triangle?) I'll give you very explicit instruction, demonstrate what I want done, and then you will be able to proceed with doing the activity. When we're done you can take these home and color them in.

Let's quickly review a couple of words that I will be using to be sure we all understand them.

6. Review terms

Draw an equilateral triangle on the board.
ASK: what is an equilateral triangle? [all sides are congruent, same length; all angles are also congruent, same size]
ASK: What is the mid-point of a line segment? [the point half way from each end]
For our exploration of the next fractal we're going to need to put a dot at the midpoint of the sides of an equilateral triangle. Where would that be? [mark it on the triangle you have on the board.]

Now hand out papers. Let's start with the side that says Sierpinski triangle.



7. Sierpinski triangle

I'll do these steps on the overhead while you do them on your paper. Don't get ahead of me for the first few steps.

1. Connect the three dots to form the sides of the triangle
2. What kind of triangle is this? [An equilateral triangle]

3. Put a dot at the mid-point of each side?
4. Connect the midpoints of each side to form more equilateral triangles.
5. How many equilateral triangles do you now have? [4]
6. Put a dot at the mid-point of each side?
7. Now connect the midpoints of each side leaving the middle triangle blank. The middle triangle is the one that does NOT touch an original vertex.
8. REPEAT steps 6 and 7 for as long as you like (can)…

Rule for this fractal: identify midpoints, connect leaving middle triangle blank, and repeat

Hint: if you are having trouble, shade the middle triangle lightly; at home you can color the triangles to make a really great piece of art

8. Sierpinski triangle Numeric Challenge (Can Be Skipped)

Can you calculate the number of triangles after each step WITHOUT counting all of them in your figure?
Create a table and observe what is changing. Start with an equilateral triangle and complete the table as you repeat the first few steps.
Note: Blank worksheet at end of this file.

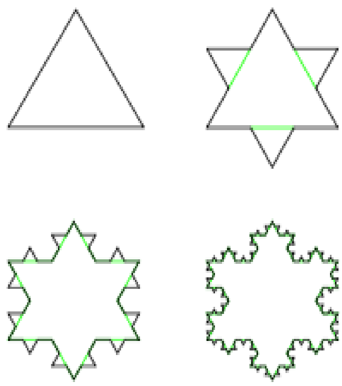
Step	Number new midpoints	Number new triangles
1	3	4
2	9	12
3	27	36
4	81	108
5	243	324

The number of midpoints is a power of 3 (3, 32, 33, 34, and so on).
The number of new triangles is always 3 times the number of new triangles in the step before because we divide each "old" triangle into three new ones included in our fractal.

9. Koch Snowflake

This fractal makes something very special, a snowflake. Use a pencil, and draw your lines lightly as you will be erasing may times for this figure.

Using the side of the handout that says The Kock Snowflake, connect the three dots to form an equilateral triangle.



Step 1: Divide each side of the triangle into three equal parts and erase the middle section.
Step 2: Replace the missing section with two line segments the same length as the section removed to form a new equilateral triangle (but with one side missing).

Repeat steps 1 and 2.

10. Koch Snowflake Numeric Challenge (Can Be Skipped)

A really interesting characteristic of the Koch Snowflake is its perimeter. Ordinarily, when you increase the perimeter of a geometric figure, you also increase its area. If you have a square and you make the perimeter larger, the area gets larger. But wait till you see what happens here!

Let's investigate the perimeter of the Koch snowflake using a table.

Step	Number of line segments	Length each segment	Perimeter = number X length
1	3	9 units	27
2	12	3 unit	36
3	48	1 units	48
4	192	1/3	64
5	768	1/9	85 1/3

Here is how the numbers were calculated.
ASK: How is the number of line segments calculated? [At each step we take the one line segment, take out a middle and add two line segments, thus changing one segment into 4 shorter segments, two of which are part of a triangle (that has one side missing). Thus the number of line segments increases by a multiple of 4 at each step.]

ASK: How is the length of each segment calculated? [At each step we divide the length of an segment by 3, so each piece is 1/3 the previous length. Also, we add two more length which are also 1/3 the previous length.]

ASK: How is the perimeter calculated? [Perimeter is just the number of segments times length of each segment.]

ASK: Interesting puzzle: Think of repeating the process for 100 times. What happens to the perimeter? The perimeter gets larger and larger! But does the area? No. We say the area is bounded by a circle surrounding the original triangle. If you continued the process oh, let's say, 100 times, the figure would have a perimeter that could go over two-thousand round-trip to the moos. But, the area would still be bounded by that circle. An "infinite" perimeter encloses a finite area... Now that's amazing!!

If the units are inches, then the perimeter is

steps	Feet	miles	roundtrips to moon
50	2,979,755	564	0
75	3,959,579,135	749,920	2
100	5,261,595,317,226	996,514,265	2,076

Sierpinski Triangle

Step	Number new midpoints	Number new triangles
1		
2		
3		
4		
5		

Koch Snowflake

Step	Number of line segments	Length each segment (inches)	Perimeter (inches)
1			
2			
3			
4			
5			

=== END OF LESSON PLAN ===