

## Exploring Maps

Notes:

- Answers to questions within the lesson are highlighted in yellow.
- Potential roadblocks:
- Front-loading vocabulary: line segment

Materials

- Exploring Map Worksheet: 2 pages
- Classroom map of US or world
- Color pencils or crayons optional

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### 1. A classroom map (if a map is available)

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Look at map in your room. (select any wall map)

How many different colors are used to color this map?

ASK: What are some of the rules used to decide how many color area needed?

[adjacent regions (that is, areas that share a common boundary) have to be different colors; diagonal regions (which share just a vertex) can be the same color]

ASK: What do you think is the smallest number of colors you will need to draw a map of a country like the US? [write some answers on the board]

Let's do some experiments to find out. This is how scientists work and discover new ideas.

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### 2. Networks (same as discussion in Traceability lesson)

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ASK: Does anyone remember what a network? [A network is a collection of points, lines and regions that are all connected to form an open or closed non-simple curve. The lines are called **paths**, the endpoint of a path or where two paths meet is called a **vertex**, and the area enclosed by the paths are called **regions**.

- The points are called VERTICES (one is a vertex)
- The lines connecting the vertices are called PATHS
- The spaces created by the paths are called REGIONS]

A **map** is another name for a network. The boundaries, or borders of states or countries on a map are generally determined by politics or national barriers (a river or mountain).

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### 3. Drawing your own map

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We are going to start by drawing our own maps and seeing how many colors we need to color it in. I'm going to draw a simple map here on the board. [draw a circle and put in the diameter – or any other closed curve following the rule “don't lift your pencil and you must end on the curve (no dangling line segments)” My simple map has two vertices, three paths and two regions (we're not counting the outside of the map).

Notice if you never cross your line, you have just two regions, an inside and outside. Boring. So, I want you to make a complex closed curve, with many regions. Don't get carried away, because we're going to have to fill these in, so make maybe four or five line crossings.

Use the letters A, B, C and so on to represent different colors and mark in the map you drew on the board. Tell the students that is what they will be doing.

Hand out papers. Start with the side that says “Exploring Maps”

Everyone draw a map WITHOUT lifting your pencil and having your end point be on some line segment[tell them to keep it simple – they can get carried away on their own time]. What is the smallest number of colors you think you need to color your map? Write some guesses on the board.

Have students use letters A, B, C etc to color their map.

ASK: How many colors are needed to color your map? [2]

ASK: Do it again! Did you get a different answer? [No, should only get 2]

ASK: Do you think this every map that is drawn this way can be colored with two colors? Why? [It is true, and the reason is that a complex closed curve drawn in this manner has exactly two regions, inside and outside which we can prove by counting number of the time you need to cross the curve to get to any region from the “outside”. Cross once, you're inside, cross twice and now you're outside]

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### 4. Coloring maps

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On the back of your pages are nine different map. What is the smallest number of colors you need to complete these maps? Use the letters A, B, C, D, and E to represent different colors. [answers below]

ASK: Why does map 7 need more colors than map 4? [In map 7, all four regions touch each other while in map 4, two of the region are not adjacent (do not share a boundary).]

You can ask about other maps and where the number of regions is related to the number of colors or if it is how the regions are touching. Like, map 6 has eight regions and needs only two color, while map 7 with only four regions needed four color.

Note that map 9 is for the western states. Be sure to color the Pacific Ocean as well as the states. Also, note that Idaho touches 6 states and Utah touches 5. Four different colors are needed to do this map.

Map	Number colors	Map	number colors	Map	number colors
1	2	4	3	7	4
2	3	5	3	8	4
3	2	6	2	9	4

There is a mathematical proof that any map in a plane needs at most four colors. Look at a map of the United States; pretty difficult to do it with only four colors. The challenge is to find a way to do it.

=== END OF LESSON PLAN ===