

The goal of this lesson is to introduce the concept of an integer, and addition and subtraction of these numbers. Sometimes there is confusion between the idea of subtraction and negative numbers. The minus sign between two numbers meaning subtractions is very different than the minus sign in front of a number meaning a negative number.

The approach used in this lesson is to introduce a set of symbols which have no number value -- a happy or sad face. Then give some rules for arithmetic and have the students work with the system. The rules are the same as the rules for integers with the difference being that here we are using happy-sad faces instead of digits. The last activity relates the happy and sad symbols to the integers on a number line.

This lesson has six parts, P1...P6. The teacher should do the intro to each part on the board and then have the students do the problems for that part. After discussing that part the teacher should proceed to the next part.

Pl: Introduction to symbols and idea of adding faces: Mathematicians like to make up number systems. They can make up any symbols they want and the rules for how they work. We're going to explore some happy and sad face arithmetic. Here what these symbol mean:


So, what do you think the answers to these addition problems are:


Mathematicians like to simplify symbols, use few strokes. We can leave off the circle and eyes to make it even simpler:

Have students do Pl

$=$


Have the students look at their problems in P1 and ask:
When adding happy faces the sum is a $\qquad$ face? (happy)

When adding sad faces the sum is a face (sad)

P2: Introduction of 0: To make our system of happy and sad arithmetic work we have to introduce a couple of rules (in math these are called axioms). What happens if we add a happy and sad face, or a sad and happy face?


We get a "big mouth" (or just another name for zero).
What happens if we add a happy to a big mouth, or a sad to a big mouth? Mathematicians have a special rule for big mouths:


We can use these rules to solve a problem like: $2 \smile+2 へ$


This diagram is trying to show that 2 happy can we expanded and 2 sad expanded and then the middle two faces combine to make a zero and similarly the outer two faces combine to make a zero, so the answer is zero (we used the rules repeatedly)

## Have students do P2

Note that the last four problems in P2 ask for the missing addend, not just the sum.

P3: Happy and sad sums: We can use tie rules in P2 to solve some problems:


Some more examples:


Have the students look at their problems in P3 and ask:
When adding, what if there are more happy faces than sad faces, is the answer happy or sad?

When adding, what if there are more sad faces than happy faces, is the answer happy or sad?

## P4: Introduction to Subtraction of happy and sad numbers.

Write the two problems in the left column below on the board. Ask: What is the answer to the subtraction problem, and what is the answer to the addition problem. Now put up the two problems on the right and ask again. What do you observe? That subtracting a happy is the same as adding a sad, or subtracting a sad is the same as adding a happy:

Have students do P4

Look at each pair of problems. Do you see that subtracting happy faces is the same as adding sad faces, and subtracting sad is same as adding happy. (Pretty neat, don't you think?)

If we think of subtraction as opposite, than opposite of happy is sad, and opposite of sad is happy. In symbols we have:

So our we can write a subtraction problem as an addition problems:


P5: General subtraction: Notice for all the problem in P4 the number of happy and sad faces is the same. We did this just so you could more easily see the relationship. What if the number of happy and sad faces isn't the same. For example,

Have students do P5

P6: Number line: We can relate our happy and sad numbers to a number line

We can represent happy and sad number on a number line as follows:


Mathematicians have names for these numbers: the integers


Integers are important number as they let us think about many things we could not if we had only the positive numbers. E.g., in a building, the floors above ground are positive while the basement floors can be thought of as negative. If you owe someone money, that is a negative amount. Temperatures below 0 degrees are negative temperatures.

You will study integers when you get to high school and learn all about their very interesting properties.
Have students do P6

Notes from 1/14/14 Math Club Teacher In-service

It was suggested that as a transition from happy-sad to integers we write the numbers with the + and - signs after the digit, then after students worked with this a while, we move to the regular integer notation; specifically,

1 happy would be 1 with a 1 sad would be 1 with a super-script plus sign

$$
1-=1^{+} \quad 1-=1^{-}
$$

These ideas need expanding into full blown transition plan showing arithmetic using this notation and then as a last step moving to traditional integer notation.

# Happy - Sad Arithmetic (Intro to Integers) Lesgone:Plan <br> (-) Happy-Sad Arithmetic : - 





